The Laney u’ Control Chart

Have you ever run into the situation where your control limits are very tight and most of the points are out of control? This can happen with u control charts, particularly if your subgroup size is large. One option to handle this is to use an individuals control chart instead of the u control chart. A better option though is to use the Laney u’ control chart. This month’s publication introduces the Laney u’ chart.

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Background Information

The classical u control chart is used to monitor data that are counts. The u control chart is based on certain assumptions. For example, it assumes that you have a Poisson distribution. There are times, though, when the u control chart gives misleading information. This is particularly true when you have large subgroup sizes.

In 2002, David B. Laney published an article in *Quality Engineering* that described a new approach to handle the situations when the sample size is large. His article is based on the p control chart. Please see our SPC Knowledge Base article, Laney p’ Control Chart, for more information. The approach is essentially the same with the Laney u’ control chart.

In this article, we will start with a set of data and use a u control chart to analyze the data. The control chart will have very tight limits with many out of control points. Then we will try the individuals control chart. This chart works better but has flat control limits – and you lose the information about the varying subgroup size. Then we will introduce the Laney u ‘ control chart and demonstrate how it solves the issue of large subgroup sizes.

The Data

A hospital group is monitoring the error rate for medicines given to patients. The group involves several hospitals. The data recorded each week is the number of patients given medicine and the number of errors. The errors can include giving the wrong dosage, giving the wrong medicine, not giving the medicine, etc. The data for 25 weeks are shown below.

Table 1: Patients and Errors Data

Week	Patients	Errors	u	Week	Patients	Errors	u
1	6566	98	0.01493	14	6598	107	0.01622
2	9671	36	0.00372	15	10029	43	0.00429

3	8129	104	0.01279	16	8192	50	0.00610
4	7757	90	0.01160	17	9417	102	0.01083
5	7880	53	0.00673	18	7130	63	0.00884
6	9102	93	0.01022	19	5455	58	0.01063
7	7201	180	0.02500	20	5946	52	0.00875
8	7940	51	0.00642	21	10222	99	0.00968
9	8486	64	0.00754	22	8154	43	0.00527
10	9858	116	0.01177	23	5114	46	0.00899
11	9226	91	0.00986	24	6256	87	0.01391
12	10496	69	0.00657	25	6466	84	0.01299
13	9427	43	0.00456				

Using the u Control Chart

The classical approach to monitoring these types of data over time is with a u control chart. The u control chart monitors the number of defects (c) in a given area where the defects have an opportunity to occur (n). In this case, the given area is the number of patients each week. The defects are errors in administering a patient's medicine each week. The value of u is given by c/n.

With u control charts, you can have an inspection unit that is made up of a certain number of patients (e.g., 1000). For this article, we will simply use an inspection unit of 1.

Each week the value of u is calculated. For week 1, c = 98 and n = 6566. This gives u = 98/6566 = 0.0149. The values of u are plotted overtime and, when enough data are available, the overall average and control limits are calculated and added to the u control chart. Figure 1 shows the u control chart based on the data in Table 1.

As you can see from the chart, 10 out of 25 points are out of control. One reason this occurs is the large subgroup size. The average and control limits, based on the assumption of a Poisson distribution, are given by:

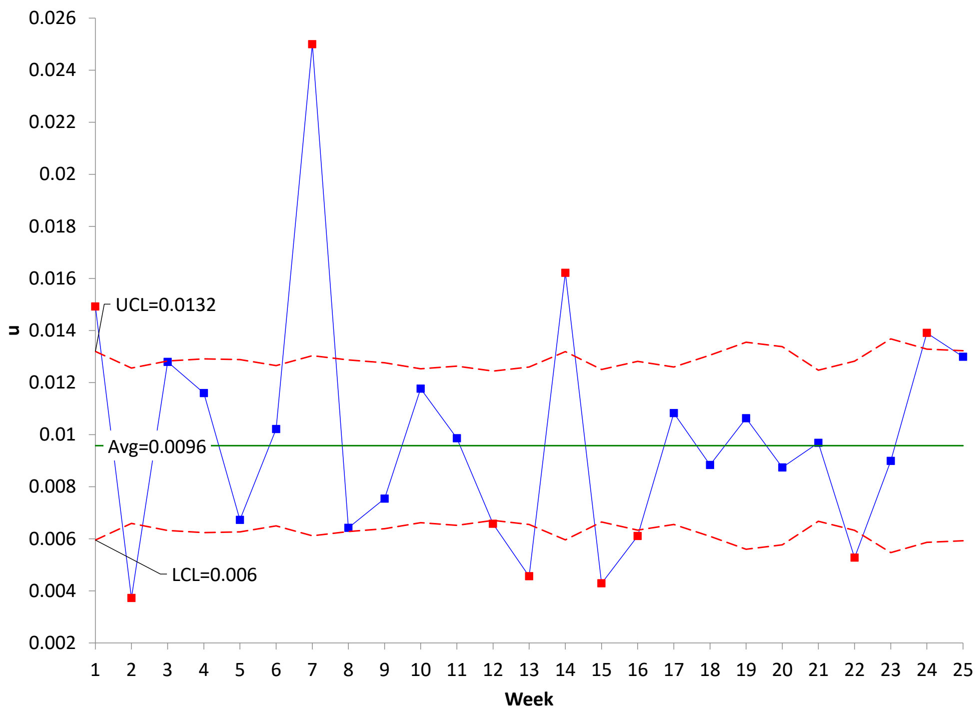
$$\bar{u} = \frac{\sum c}{\sum u}$$

$$UCLu_i = \bar{u} + 3\sqrt{\frac{\bar{u}}{n_i}}$$

$$LCLu_i = \bar{u} - 3\sqrt{\frac{\bar{u}}{n_i}}$$

The u control limits vary as the subgroup size varies. Also, the control limits become very tight as n_i gets larger – the $LCLu_i$ and the $UCLu_i$ get closer together. This is why so many points are beyond the control limits. Let us see what impact using an individuals control chart will have.

Figure 1: u Control Chart for Patients/Errors Data



Using the Individuals Control Chart

In the past, the individuals control chart has been used to address the issue of large subgroup sizes with attributes charts. The individuals control chart is really two charts, the X control chart and the moving range chart. Each individual u value is plotted on the X chart while the range between consecutive u points are plotted on the moving range chart.

Once enough data are available, the overall average and X control limits are added to the X chart, and the average moving range and moving range control limits are added to the moving range chart. Figure 2 is a plot of the X chart for the data in Table 1. We will not show the moving range chart here.

This control chart looks entirely different from the u control chart in Figure 1. Instead of a lot of out of control points, there are none! The control limits with the X control chart are constant. They are given by:

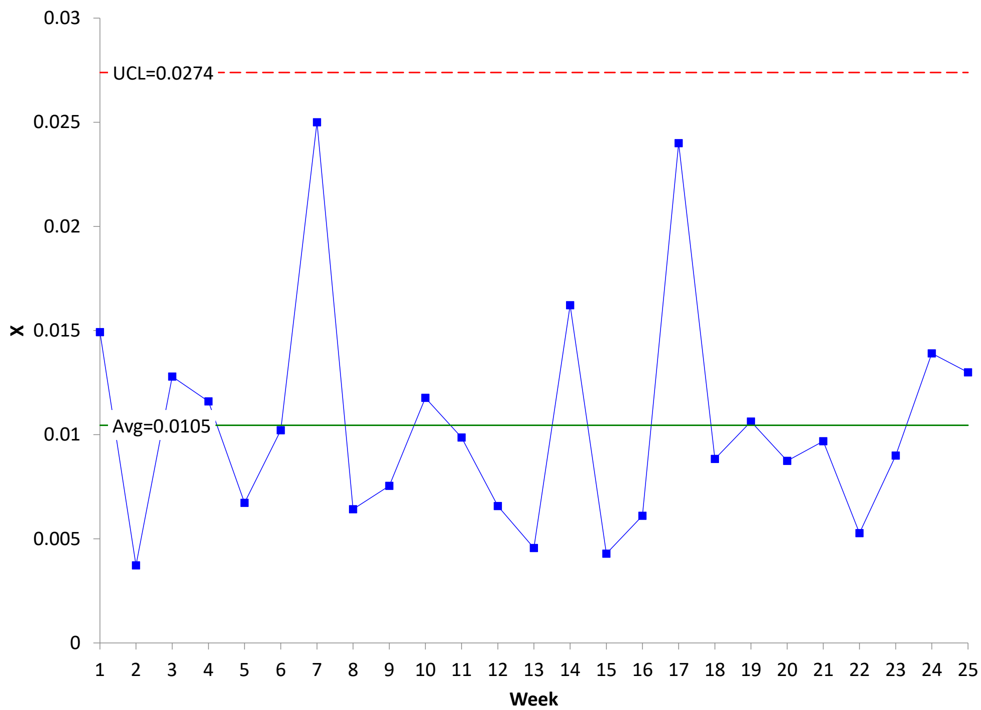
$$\bar{X} = \frac{\sum X}{k}$$

$$UCLx = \overline{X} + 2.66\overline{R}$$

$$LCLx = \overline{X} - 2.66\overline{R}$$

where X is the individual u values and k Is the number of points. Please see our SPC Knowledge Base article, Individuals Control Charts, for more information.

Figure 2: X Chart for Patients/Errors Data



The control limits make a lot more sense. There is nothing wrong with this approach – using the individuals control chart. There is only one criticism about the individuals control chart in this situation – it does not account for varying subgroup sizes. Often that will not be an issue, but there are times when it can be – particularly with large subgroup sizes. The Laney u' control chart takes care of this potential problem.

Using the Laney u' Control Chart

The Laney u' control chart, like the other two, plot the values of u over time. The overall average is calculated using the same equation as for the u control chart above. Then things get a little different. The Laney u' control chart is produced by combining z values with the individuals control chart.

A z value is calculated for each point. A z value tells you the number of sample standard deviations between the point and the overall average. The z value for point i is given by:

$$z_i = \frac{u_i - \bar{u}}{\sqrt{\frac{\bar{u}}{n_i}}}$$

This is done for each u value. The results are shown in Table 2.The next step is to calculate the moving range between consecutive z values – just like we do with X values in the individuals control chart. The moving ranges are shown in Table 2. The average moving range is then calculated.

$$m\bar{R} = \frac{\sum mR}{k-1}$$

Table 2: Calculations for the Laney u' Chart

Week	u	z	mR	UCL	LCL
1	0.01493	4.42997		0.02497	-0.00582
2	0.00372	-5.88223	10.31220	0.02226	-0.00311
3	0.01279	2.96505	8.84728	0.02341	-0.00426
4	0.01160	1.82421	1.14084	0.02374	-0.00459
5	0.00673	-2.58514	4.40935	0.02363	-0.00448
6	0.01022	0.62583	3.21097	0.02265	-0.00350
7	0.02500	13.37281	12.74698	0.02428	-0.00513
8	0.00642	-2.87061	16.24343	0.02358	-0.00443
9	0.00754	-1.91458	0.95603	0.02312	-0.00397
10	0.01177	2.22355	4.13813	0.02214	-0.00299
11	0.00986	0.28250	1.94104	0.02257	-0.00341
12	0.00657	-3.14264	3.42514	0.02175	-0.00260
13	0.00456	-4.97519	1.83255	0.02243	-0.00328
14	0.01622	5.51294	10.48813	0.02494	-0.00578
15	0.00429	-5.41180	10.92473	0.02203	-0.00288
16	0.00610	-3.21148	2.20032	0.02336	-0.00421
17	0.01083	1.24541	4.45688	0.02243	-0.00328
18	0.00884	-0.63830	1.88371	0.02435	-0.00520
19	0.01063	0.79766	1.43596	0.02647	-0.00732
20	0.00875	-0.65424	1.45190	0.02576	-0.00661
21	0.00968	0.11300	0.76724	0.02192	-0.00277
22	0.00527	-3.96996	4.08296	0.02339	-0.00424
23	0.00899	-0.42438	3.54558	0.02702	-0.00787
24	0.01391	3.50071	3.92509	0.02535	-0.00620
25	0.01299	2.80658	0.69413	0.02509	-0.00594

k is the number of data points. The moving range is 4.79419. The next step is to estimate sigma for the z values in Table 2. This is done by dividing the average moving range by 1.128, again just like in the individuals control chart. Sigma for the z values is:

$$\sigma_z = m\overline{R}/1.128 = 4.25017$$

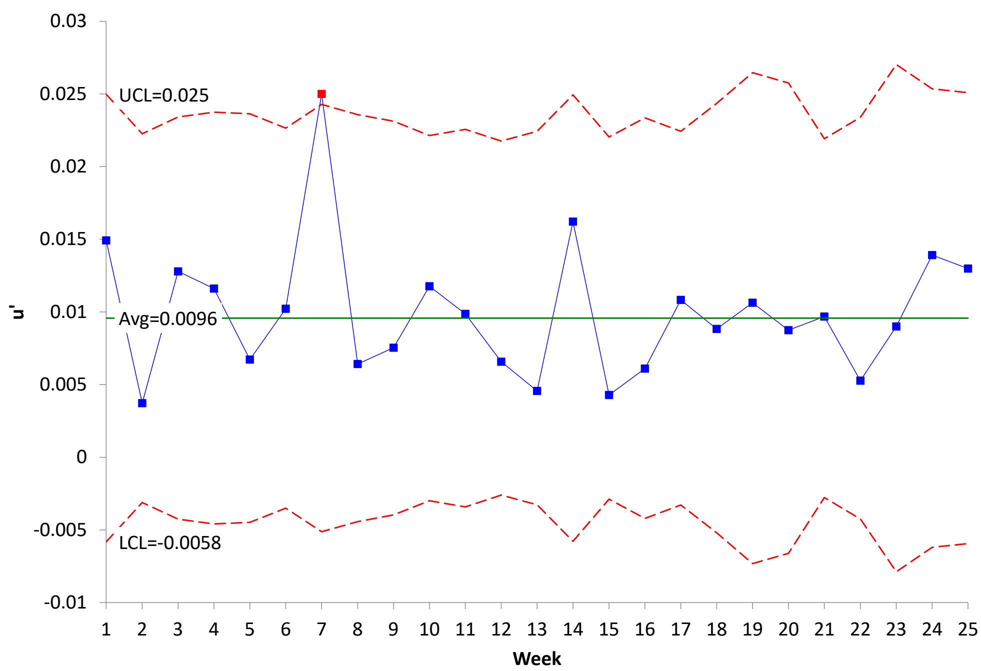
This value of σ_z is used to adjust the u control chart limits, as shown below.

$$UCLu'_i = \bar{u} + 3\,\sigma_z\sqrt{\frac{\bar{u}}{n_i}}$$

$$LCLu'_i = \bar{u} - \,3\,\sigma_z\sqrt{\frac{\bar{u}}{n_i}}$$

The control limits are shown in Table 2 as well. Plotting the u values, the overall average, and the control limits give the Laney u' control chart shown in Figure 3.

Figure 3: Laney u' Control Chart for Patients/Errors Data



Take a close look at Figure 3. A couple of items to notice. First, the control limits vary because the subgroup size (number of patients) varies each week – just like the u control chart. Now note that there is one out of control point with Laney u' control chart while the u control chart had many. The individuals control chart did not have any. The reason the Laney u' control chart picks it up is that it is accounting for the varying subgroup size – which the individuals control chart does not. In this situation, the Laney u' control chart is a better than the individuals control chart.

Summary

This publication introduced the Laney u' control chart. This control chart is especially useful in handling large subgroup sizes that lead to very narrow limits on the traditional u control chart. It does this by combining the calculation of z values with the X control chart as shown above. The Laney u' control chart is a better choice when you have large subgroup sizes.

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Thanks so much for reading our publication. We hope you find it informative and useful. Happy charting and may the data always support your position.

Sincerely,

Dr. Bill McNeese
BPI Consulting, LLC