Rational Sampling and the Individuals Control Chart

Over the years, the individuals control chart (X-mR) has become more and more popular, replacing the $\overline{X}$-R chart as the control chart of preference in many situations. In fact, some have declared that the X-mR chart is the only chart you ever need to use. The X-mR chart has replaced the attributes control charts in most cases.

One advantage of the X-mR chart is that it is simple to explain and implement. After all, you simply plot the individual result on the X chart and the moving range between consecutive samples on the mR chart.

So how do most people use the X-mR chart? They take their data and put it in a software package. The software package generates the two control charts (the X and the mR) with averages and control limits, highlighting any out of control points. So simple.

It sounds simple. But sometimes you need to think. Is my method of sampling the process capturing the routine process variation between samples? How do I know? Is the data collection so frequent that the routine process variation is not being captured between samples? What does that do to the control chart?

Lots of questions – more than we normally think about. One key to setting up an X-mR chart correctly is to understand rational sampling. This publication explores the concept of rational sampling and how it impacts X-mR charts. Satisfying the rational sampling concepts becomes more difficult as the frequency of data collection increases.

In this publication:

Introduction

There are two concepts that are important in determining how effective your sample procedure is for the process you want to monitor: rational subgrouping and rational sampling. Rational subgrouping is one key in the effective use of averages control charts,

such as the $\overline{X}$-R chart. It is based on sampling your processes in a way that explores the variation you are interested in.

You set up the subgroups so that the variation within a subgroup is minimized, and the variation between subgroups is maximized. This makes the $\overline{X}$ chart do the work of finding special causes of variation. We have five articles on rational subgrouping in our SPC Knowledge Base: rational subgrouping.

The X-mR chart does not have subgroups. You are plotting individuals values on the X chart and moving range values on the mR chart. So rational subgrouping does not directly apply, although you still want to sample your process in a way that explores the variation you are interested in. With the X-mR chart, you need to think about rational sampling.

It used to be that people were taught you only used the X-mR chart if you had infrequent data. You used the $\overline{X}$-R chart if you had frequent data. Some are now using the X-mR chart with frequent data. The conditions for rational sampling are easier to satisfy when you have infrequent data, less so when you have frequent data.

Let's explore rational sampling in some more detail.

Rational Sampling

Dr. Donald Wheeler is an expert when it comes to control charts. Over the years he has written numerous times about rational subgrouping and rational sampling. He recently published an article on the sample frequency and process behavior charts. In it, he states that the two requirements of rational sampling for the X-mR chart are:

1. Successive individual values must be logically comparable.
2. The differences between successive values must logically capture the routine process variation.

What do these two things mean? Dr. Wheeler uses an example where a temperature is measured in a process every second. Dr. Wheeler shows examples of X-mR charts based on measuring the temperature every 4, 8, 16, 32, 64, and 128 seconds. I never would think about using an individuals control chart with a point every 4 seconds. But his purpose in doing this was to show what happens when either of the rational sampling statements are not met. It is all about variation.

What he showed was that the more frequent sampling leads to a situation where the routine variation was not being captured within the time between samples, and this led to

false out of control points. Let's review how the moving range between consecutive samples captures the variation in a process.

Variation and the Individuals Control Chart

With the X-mR chart, you plot the individual values on the X chart and plot the moving ranges between consecutive points on the mR chart. Suppose you are monitoring the level of a contaminant in a process stream. You take one sample per hour and measure the level of the contaminant. You are going to plot the results on the X-mR chart to monitor the level over time.

Does this sampling plan take into account rational sampling? Let's look at the two components of rational sampling.

- *Successive individual values must be logically comparable*.
- In this example, we are comparing successive contaminant levels at one point over time. The successive samples (one taken each hour) are logically comparable. So, this meets the first requirement. This will usually be the case as long as you compare successive samples, one point at a time. It is also good to have the time between samples be the same, e.g., each hour.
- *The differences between successive values must logically capture the routine process variation*.

  This requirement is more challenging to meet. It will be met if the time between samples is sufficient to contain the routine variation in the process. Is one hour enough? You must make that decision based on your knowledge of the process. In this example, an hour may well be enough.

The moving range is the key. The moving range measures the variation between consecutive samples. Suppose you take a sample and measure the contaminant level. The result is X_1. The next hour, you take another sample and measure the contaminant level. The result is X_2. X_1 and X_2 may be identical but most likely are not.

The sources of variation in the process are what cause the consecutive values to be different. What are the potential sources of variation? Of course, this depends on the process, but sources of variation can include sampling error, measurement error, time of day, temperature, humidity, etc. It can be and is a long list.

The control limits for the X chart are calculated using the average moving range. To meet the requirements for rational sampling, the moving ranges between consecutive values must be logically comparable. If the moving ranges contain different sources of variation,

then the moving ranges will not be comparable over time. It will be like comparing apples to oranges.

The difference between consecutive values should represent common causes of variation, the normal variation in the process. These differences are then used on the X chart to determine the control limits and filter out the normal variation on the X chart.

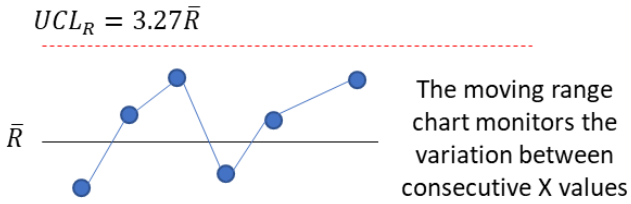
The control limits for the moving range chart are given by:

$$\bar{R} = \frac{\sum R}{k-1}$$

$$\text{UCLr} = 3.268\bar{\text{R}}$$

where $\bar{\text{R}}$ is the average moving range, k is the number of samples, and UCLr is the upper control limit for the mR chart. The variation in the mR chart is shown in Figure 1.

Figure 1: mR Control Chart Variation



Again, the mR chart is looking for consistency between the moving range values. $\bar{\text{R}}$ is then used in the calculation of control limits for the X chart:

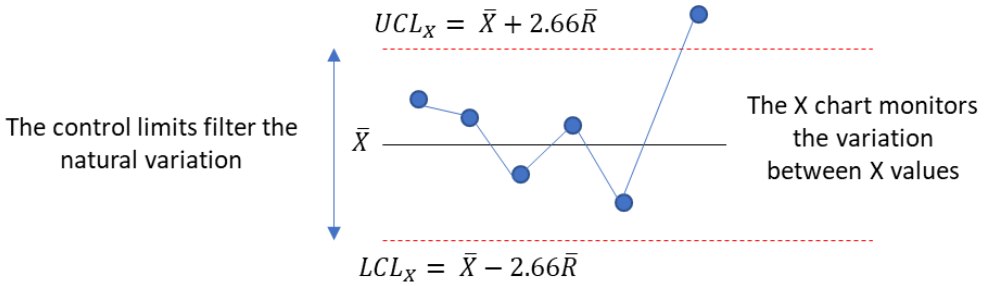
$$\bar{X} = \frac{\sum X}{k}$$

$$\text{UCLx} = \bar{\text{X}} + 2.66\bar{\text{R}}$$

$$\text{LCLx} = \bar{\text{X}} - 2.66\bar{\text{R}}$$

where $\bar{\text{X}}$ is the overall average of the samples and UCLx and LCLx are the upper and lower control limits for the X chart. Figure 2 shows how the natural variation is filtered by the control limits.

Figure 2: X Control Chart Variation



The moving range plays a key role in setting the control limits for both charts. You want to sample your process, so each moving range contains the routine variation in the process. This basically gets down to a time issue:

> *"How long do I need to wait between samples to be sure that the sources of routine variation have had the opportunity to be present in the process?"'*

In other words, how rapidly does your process change?

Should I Use the X-mR Chart with Frequent Data?

As the frequency of data collection increases, the use of the X-mR chart becomes more complicated. This is because it becomes harder to meet the two requirements for rational subgrouping, in particular, the second one: differences between successive values must logically capture the routine process variation.

So should you use the X-mR chart with frequent data? Of course, this depends on your definition of frequent data. X-mR charts work well with data that is daily, weekly or monthly when you are talking about time between samples. They also work well when you compare batches, shifts, etc. With these types of data, the X-mR chart will often meet the requirements of rational sampling.

But when your data is collected very frequently (e.g., every second, every minute), the X-mR chart begins to have problems with rational sampling. There is most likely no process where the moving range between consecutive samples taken every second captures the routine process variation. Not to mention the huge number of points you have if you take a sample each second. Sampling each second is 3,600 points per hour.

I believe the answer to this question is no if you are talking about very short periods of times between samples, like seconds or minutes. The question you have to answer is what is frequent for me. Plotting a point each second or minute is way too frequent.

What should you do if you have very frequent data and still want to use a X-mR chart? One option is to plot a point with a set time interval, such as a point every minute, every five minutes, etc. This can still be a lot of data on the X-mR chart. You will probably have to do this trial and error to see what the best time interval is for your process and to decide if it meets the requirements of rational sampling.

In reality, the better approach with very frequent data is to look at subgrouping the data, again to explore the variation you are interested in. You might use an $\overline{X}$-R or $\overline{X}$-s chart with the subgrouping . Another option is to set your time period (e.g. five minutes) and each 5 minutes calculate the average and standard deviation for the values over that 5 minute period. Then use an X-mR chart for the average and an X-mR chart for the standard deviation. An example of doing this is in our SPC Knowledge Base article Control Charts and Data Overload.

Summary

This publication examined how rational sampling impacts X-mR charts. The X-mR chart should meet the requirement of rational sampling:

1. Successive individual values must be logically comparable
2. The differences between successive values must logically capture the routine process variation.

Rational sampling works well when data are collected daily, weekly or monthly. But as data collection becomes more frequent, it is more difficult to meet the requirements of rational sampling, in particular the second item. For very frequent data (such as every second, every minute), you probably need to look at other options than the classical X-mR chart.

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Thanks so much for reading our publication. We hope you find it informative and useful. Happy charting and may the data always support your position.

Sincerely,

Dr. Bill McNeese